CASE STUDY 4: MECHANICS

MODELING AND SIMULATION

NOVEMBER 5, 2008

Overview

In this case study, you will use Newtonian mechanics to model objects in motion. The first option focuses on the simulation of planetary motion in a binary star system; the second option looks at the design of skateboard ramps; and the third option is a "roll your own" case with some minimal guidelines.

These options require different disciplinary knowledge. In the binary star case, you will likely use point mass approximations. Torque and angular momentum are potentially useful but not really necessary. The second option calls for the torque and angular acceleration of rigid bodies, but you can avoid dealing with rigid bodies that both rotate and translate. In the third option, some of the suggestions require that you deal with rigid bodies that both translate and rotate.

Interim Deliverables

Although each project will be different, we expect the following interim deliverables for all options.

Question and Model: Using butcher paper, prepare a draft poster that presents your question and proposed model. This is due no later than the beginning of class on *Friday, 14 November*.

Key Simulation Results: Prepare a draft poster, again on butcher paper, showing your key simulation results. Be sure you have some validation, as well as results that begin to answer your question. This is due no later than the beginning of class on *Friday, 21 November*.

Draft of Entire Poster: Prepare a draft of your entire poster using butcher paper. This is due no later than the beginning of class on *Friday, 5 December*.

Draft Presentation Script: Create a rough script for your presentation. Who is saying what? What are they referring to on the poster when they say it? What are the key points that they should highlight? You might make this script with multiple numbered post-it notes, and attach the post-its to your poster rough draft, so that it's clear how the script interacts with the poster. However you choose to do it, have a draft script posted and ready to discuss by the beginning of class on *Friday, December 5*.

Final Recorded Presentations

Your final presentations of posters are due during the finals period. A signup sheet for these presentations will be available after Thanksgiving. When you come to your presentation time, you should bring the following with you:

Final Poster: Create a final version of your poster. You may use the poster printer for this, but you are not required to do so—a nicely produced cut-and-paste poster is just fine. This poster must be completed in time for you to make your presentation video; bring it with you at your scheduled presentation time.

Presentation Video: Make a video of your presentation, and store it on your laptop. The video should be very simple—we want a fixed frame video of you and your partner doing your poster presentation. Your video can be no more than six minutes in length (unless you are a team of three, in which case it cannot exceed nine minutes). Bring a laptop with the video stored on it to your scheduled presentation time.

Presentation Assessment Sheet: Bring these sheets with you to your scheduled presentation time.

Celestial mechanics

The question of whether a relatively low-mass planet can survive for long in a binary star system is of considerable scientific interest; a paper by Holman and Wiegert[1] details the results of relevant numerical simulations.

Holman and Wiegert considers the case of a low-mass planet in orbit about binary stars of mass ratio μ which are in elliptical orbit of eccentricity e. In the last section of the paper, the authors suggest some possible future projects. One of them involves the "instability islands" mentioned on page 626; the other involves giving the planets initial non-zero eccentricities.

If you choose this option, you will want to spend some time reading the paper in depth[2]. You will the develop a model for the system, implement a simulation, and validate it both against known behavior of simpler systems (e.g., is there a limiting case in which this looks like a single star system?) and against some of the results in the paper (e.g., some of the results in Tables 2 and 3). Finally, you might use your simulation to begin to investigate one of the issues raised at the end of the paper.

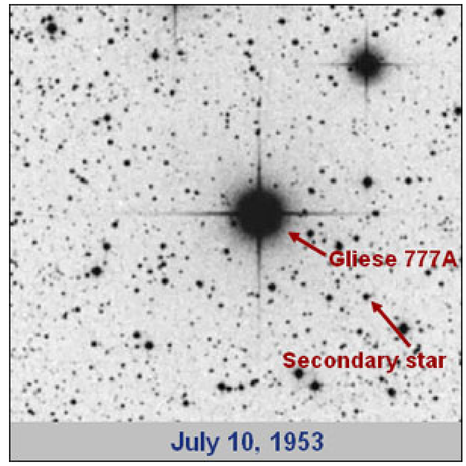

Figure 1: A binary star system from from planetquest.jpl.nasa.gov.

[1] *The Astronomical Journal*, Volume 117, Pages 621–628, Year 1999

[2] It is, not unlike neutron star, rather dense.

Skateboarders and Ramps

Skateboarding includes a variety of interesting and rather counter-intuitive tricks, like the Ollie, in which the skateboarder is somehow able to, while standing on top of the skateboard, cause the board to launch itself vertically in the air (check it out on *YouTube*). But one thing has bothered us consistently about skateboard competitions – they almost always involve fixed obstacles, like the boring skateboard ramp shown in Fig. 2. Wouldn't the life of skateboarders everywhere be better if the ramp could move?

In this project, you are asked to to design a new ramp which pivots. This design will allow a rider to approach the ramp on a flat surface and then coast up the ramp; if they are going fast enough, the ramp will rotate and they will gracefully ride down the rotating ramp (right?).

A successful design will include the following:

- A simple model that allows an estimate of the length of ramp and mass of ramp for a typical rider approaching at a typical velocity.
- A dynamic simulation of the system, based on a model with enough detail to predict the minimum velocity a rider needs to get over the ramp and down the other side.

Of course, if you'd rather model something else in the world of skateboarding, we're on for that as well – just be sure you are doing some work with the model!

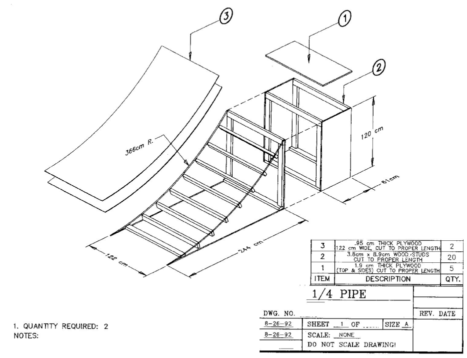

Figure 2: A compelling picture of a lame skateboard ramp.

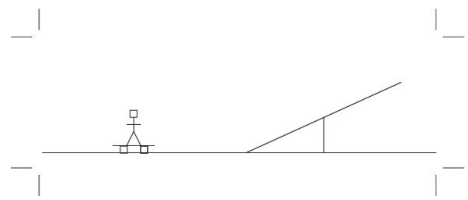

Figure 3: A lame picture of a compelling skateboard ramp.

Create Your Own Case Study

If neither of the previous options suits you, you have the option of creating your own case study.

You can study any kind of physical system that interests you, with the restriction that the primary model you use should be based on a system of second-order differential equations.

Here are some ideas to help you get started:

Figure 4: Brachiation from `www.gibbons.de`.

Brachiation: Gibbons are able to move through trees using very little power and at extraordinary speed (35 miles an hour!) by brachiating—swinging only from their arms, spending some portion of their time free-falling, and some portion acting like a pendulum. Can you simulate this gait? Can you optimize it? Note: You will find a variety of fairly advanced approaches online to brachiation (e.g, a five link brachiation simulation that attempts to capture the motion of the upper arm, lower arm, and torso in 3D). If you have time for a PhD, this would be a good approach.

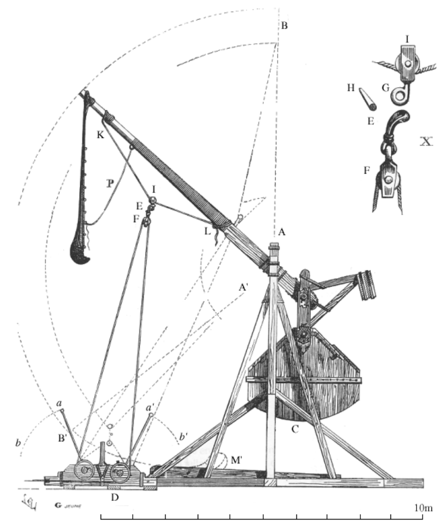

Figure 5: Trebuchet from `en.wikipedia.org/wiki/Trebuchet`.

Swing up a pendulum: There are a variety of fun control problems involving getting a pendulum to do what you want it to do. One relatively simple one is this: imagine you have a pendulum, with a motor acting as the pivot. The motor's torque output is insufficient to invert the pendulum in one go. Can you control the motor in such a way that it inverts the pendulum (and keeps it there)? Warning: there are lots of YouTube videos of the swing-up task for rotary pendulums. This suggestion is simpler than these!

The Segway and the Broomstick: More challenging from a physics perspective (although not, perhaps, from a control perspective) are problems in which you try to keep a pendulum with a movable pivot inverted; e.g., the Segway, or an inverted pendulum on a cart.

Siege Engines: Catapults, onagers and trebuchets, oh my! If this is your cup of tea, there's a great 2005 paper by Mark Denny, "Siege Engine Dynamics" in the *European Journal of Physics*. You can take it from there[3].

Tiger Woods Problem: A golf ball[4] hit with backspin generates lift, which might increase its range, but the energy that goes into generating spin probably comes at the cost of lower initial velocity. Write a simulation of the flight of a golf ball and use it to find the launch angle and allocation of spin and initial velocity (for a fixed energy budget) that maximizes the horizontal range of the ball in the air.

[3] No, you may not build one to throw a Volkswagen.

[4] See `http://en.wikipedia.org/wiki/Golf_ball`.

Physics of Sports: By design, many sports take place in domains where different kinds of motion, forces, material properties, etc. come into play (literally). Choose a sport and see if you can use a dynamic model to answer a relevant question. One resource you might find useful is *The Physics of Baseball*[5].

[5] Robert K. Adair, Harper Paperbacks, 3rd Edition, 2002.

If you don't like any of these suggestions, you are free to propose your own.