CASE STUDY 1

MODELING AND SIMULATION

Population models make the world go around.

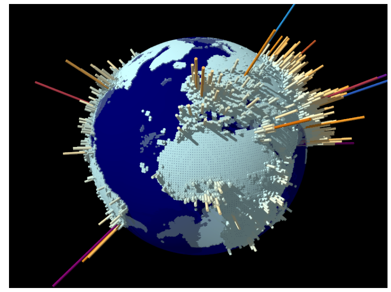

Figure 1: Population density from `flickr.com/photos/arenamontanus/375127836/`.

Case Study 1: Population Dynamics

Please choose one of the following three cases. The first case involves student demographic data at a small New England college with a rather interesting set of students. The second case involves the life cycle of a small flour beetle called *Tribolium*. The third case concerns the demographics of primary education in developing countries, with a focus on the question of how best to achieve universal primary education in Uganda.

Each case expects you to do something a little different. For example, in the *Student Pipeline* case you are asked to create a model for student enrollment and use numerical simulations to make predictions concerning the number of students on campus over the next several years. In the *Tribolium* case you are provided with the reference to a scientific paper, asked to implement the model presented there, reproduce some of the key findings, and propose and answer some questions about this model. In the *Uganda* case you are asked to create a model of pupil progression through Uganda's educational system, and use the model to make recommendations about where aid agencies and policy makers might most effectively focus their efforts.

While each case has more or less guidance about what you should *do*, there are certain expectations that are common to each. You are expected to formulate and/or implement a model using MATLAB. You are expected to work with a teammate. You are expected to create a set of *deliverables*[1]. The deliverables for each case will probably be different. However, the required deliverables are meant to help us interact with you and to help you develop skills in oral and visual communication.

By our next meeting you should have chosen (with your teammate) a case study to pursue.

[1] A recent addition to the Oxford dictionary meaning *a thing able to be provided, especially as a product of a development process.*

The Student Pipeline

How many students can we enroll?

Every year the President's Cabinet[2] meets to choose a target for the number of students who will enroll in the freshman class the following year.

The members of the Cabinet consider several goals in choosing their target: one is to admit as many students as possible, subject to the constraint that the number of students on campus never exceeds the number of beds[3] in the residence halls; another is to minimize variation in class size from year to year.

To make this decision, the Cabinet needs to predict the number of students on campus over the next several years. This prediction is difficult because not all students who enroll are in residence for eight consecutive semesters. Some take a leave of absence or study away, some leave the college altogether, and some take more than eight semesters to graduate.

That's where you come in. We want you to help by building a model of the flow of students through the college, writing a computer simulation of your model, and using it to evaluate alternative strategies for choosing the number of students to enroll.

Figure 2: Student pipeline?

[2] The President of Olin College, that is.

[3] Intermittent sharing is not a sustainable solution.

Data

We have collected some data[4] that might help you formulate this problem and build your model. There is a lot of information here, so you will have to decide which factors to consider and which can be simplified or ignored.

Whenever you work with demographic data, keep in mind that the statistics reflect the lives of real people. Please approach this project with appropriate discretion and sensitivity.[5]

The following table shows the number of students in residence at Olin during the last 12 semesters. The rows indicate the classification of the students:

FF: First-time freshman, an incoming student who has not attended any postsecondary institution after graduating from high school (except for summer immediately following).

FR: "Returning" freshman, a student who has earned 0–30 credits at a postsecondary institution (including Olin), but who is not a first-time freshman[6].

SO: A student with sophomore standing (30–60 credits).

JR: A student with junior standing (60–90 credits).

SR: A student with senior standing (90–120 credits).

SP: An exchange student or other special registration.

[4] Many thanks to our Registrar, Linda Canavan.

[5] Don't be a jerk.

[6] Second time's the charm!

	0203FA	0203SP	0304FA	0304SP	0405FA	0405SP	0506FA	0506SP	0607FA	0607SP	0708FA	0708SP
FF	43		56		49		70		85		79	
FR	32	59	22	65	30	59	14	76	10	86	7	77
SO		16	71	59	68	74	63	54	75	78	85	81
JR				23	52	62	66	70	55	51	64	56
SR					1	5	65	67	70	77	60	67
SP									2	1	3	6
Total	75	75	149	147	200	200	278	267	297	293	298	287

The following is an email from the Dean of Student Life, Rod Crafts, detailing the availability of beds[7]:

[7] Thanks for your help, Uncle Rod!

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from    Roger (Rod) Crafts <Roger.Crafts@olin.edu>
to  Modeling and Simulation Faculty
subject beds?

Hi All,

This may be more complicated than you need or want.

East Hall has 182 bedspaces, minus 2 bedspaces for the Admission guest
room = 180 available bedspaces

West Hall has 181 bedspaces, minus 46 bedspaces for team rooms, gym,
SCOPE offices, and SCOPE office furniture storage = 135 available
bedspaces

Beyond the constraints listed above:

- we have a few students with medical documentation for single rooms,
so we end up with empty beds in doubles for this reason

- if we have an uneven number of men or women in the
entering class, we end up with an empty bed in a double for this
reason

Obviously, these factors further reduce our available capacity.

We could accommodate 20 additional students by reducing the allocation
of rooms for SCOPE or a total of 36 additional students by eliminating
SCOPE offices in West Hall.

In the past, all students have lived on campus with the exception of
two during 2006-07 and five during 2007-08.  Under the new
Trustees policy, students may only live off-campus under exceptional
circumstances.

I hope this helps.

Rod
```

Deliverables

The final deliverable for this case study is a briefing sheet, which is a 1–2 page report that summarizes your recommendations to the Cabinet. You should explain and justify your recommendation at a level of detail that would allow a Vice President of the college to understand what you did and to form an accurate impression of the accuracy and reliability of your results. For example, it would be appropriate to include quantitative results from your simulation, but you should not include MATLAB code[8]. This document must be turned in by 9AM on *Wednesday, September 17*.

[8] Numbers, yes. MATLAB, no.

In addition, we strongly recommend that you prepare the following intermediate deliverables:

Model Diagram and Explanation: Create a clearly annotated diagram for your model. The diagram should include a caption that describes the parameters and how you determined or will determine their values. You should plan to post this diagram in your workspace no later than the end of class on *Monday, September 8*.

Script: Write a MATLAB script that implements the model. The script should be well commented and easy to read. You should plan to post your working code in your workspace no later than the end of class on *Wednesday, September 10*.

Validation and Preliminary Results: Create one or more figures with descriptive captions that demonstrate the extent to which your model and simulation agree with the available data. Also create one or more figures that present your preliminary results. These should be completed and posted in your workspace no later than the end of class on *Monday, September 15*.

The Life-Cycle of Tribolium

Introduction

The red flour beetle, *Tribolium castaneum*, is a common pest which attacks stored products such as flour[9] and cereals. These beetles can multiply into large populations and destroy products in warehouses and pantries alike. They are about 1/2 cm in length and can fly. Female beetles each lay approximately 2 to 3 eggs per day for a period of up to 8 months. Within 5 to 12 days these eggs hatch into larvae, and after the larval period (which can last anywhere from 20 to 100 days), the fully grown larvae transform into pupae. After 8 days an adult emerges from the pupae and can live for about 3 years. More information about this and other pests can be found at `http://www.the-piedpiper.co.uk`.

Figure 3: Tribolium castaneum.

[9] By law wheat flour may contain up to 75 insect fragments per 50 grams.

In 1995 a group of researchers published a letter in the prestigious journal *Nature* on the results of an investigation of the population dynamics of the red flour beetle [10]. In this joint theoretical and experimental study the authors test the hypothesis that changes in demographic parameters cause predictable changes in the nature of population fluctuations. They predict mathematically, and then observe experimentally, that manipulating the adult mortality rate changes the dynamics from stable equilibria to periodic cycles to aperiodic oscillations.

[10] R.F. Constantino, J.M. Cushing, B. Dennis, and R.A. Desharnais. Experimentally-induced transitions in the dynamic behavior of insect populations. *Nature*, 375(6528):227–230, Jan 1995.

Their model (described more fully in the paper and references therein) takes the form of three coupled difference equations,

$$
\begin{aligned}
L_{t+1} &= bA_t \exp(-c_{ca}A_t - c_{cl}L_t), \\
P_{t+1} &= L_t(1-\mu_l), \\
A_{t+1} &= P_t \exp(-c_{pa}A_t) + A_t(1-\mu_a),
\end{aligned} \tag{1}
$$

for the number of larval L_t , pupal P_t, and adult A_t red flour beetles at time t. The values of the six demographic parameters in the model, including the adult mortality rate μ_a, were obtained by fitting the model data to experimental data obtained in a laboratory setting. After calibrating the model the authors went on to manipulate the adult mortality rate experimentally[11] and demonstrated that the resulting population dynamics agreed with the predictions of their model.

[11] They killed them.

Deliverables

In this case study your task is to create a simulation of the population dynamics of a pest (either Tribolium or some other pest), and to use your simulation to investigate the effects of varying one or more demographic parameters. The final deliverable for this case study is a short scientific memo titled "Computational Investigation of Population Dynamics in [*Insert pest name here*]".

Should you choose to investigate Tribolium, your memo should be written for an audience that is familiar with Constantino's work, and should somehow extend that work by investigating the variation of a different demographic parameter than is investigated in the paper. Should you choose to investigate another pest, your memo should present your model as well as results from the implementation of your model. Note that whatever you choose to do, your memo should *not* include MATLAB code. This document must be turned in by 9AM on *Wednesday, September 17*.

In addition, we strongly recommend that you prepare the following intermediate deliverables:

Model Diagram and Explanation: Create a clearly annotated diagram for the model for Tribolium. The diagram should include a caption that describes the demographic parameters. You should plan to post this diagram in your workspace no later than the end of class on *Monday, September 8*.

Script: Write a MATLAB script that implements the Tribolium model. The script should be well commented and easy to read. To demonstrate that your simulation is consistent with the paper, please produce the following figures:

1. A figure representing the dynamics of the SS strain over 36 weeks for $\mu_a = 0.04$, 0.50, and 0.96. (see figure 3 in the paper)
2. A figure representing the bifurcation diagram for the SS strain as a function of adult mortality rate μ_a. (see figure 4b in the paper)

These figures should include sensible captions, and should be completed and posted in your workspace no later than the end of class on *Wednesday, September 10*. Please post your code at this time as well.

Preliminary Results: Whether you are doing further investigation of Tribolium or a new investigation of a different pest, please be sure to post some preliminary results (e.g., the figures you plan to include in your memo) no later than *Monday, September 15*.

Educational Reform in Uganda

Introduction

One of the United Nations' Millenium Development Goals is universal primary education by 2015 [12]. There has been substantial progress toward this goal: in general, increasing numbers of children are entering primary school, and the gender gap in school attendance is shrinking world-wide. However, retention of students in primary school remains a significant problem: according to the Education and Policy Data Center, world-wide, 34% of all children will not complete primary school—12% because they do not begin primary school, and 22% because they drop out at some point during the course of primary school.[13]

The situation in Uganda mirrors world-wide trends. As can be seen in Fig. 2, the latest demographic data suggests that only about 50% of pupils in Uganda ever reach the 5th grade, and that of those who do, only perhaps 75% are able to read a sentence. In short, attrition combined with ineffective learning lead to relatively poor outcomes in Uganda. Aid agencies and the government of Uganda would of course like to improve these statistics—but as with most things in life, resources are limited.

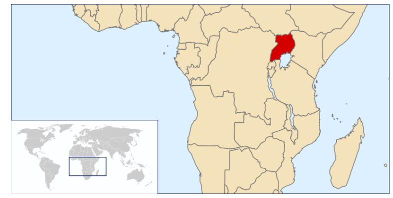

Figure 4: Oh Uganda, Land of Beauty.

[12] Inter-Agency Expert Group. The millenium development goals report. Technical report, United Nations, New York, New York, 2007.

[13] G. Ingram, A. Wils, B. Carrol, and F. Townsend. The untapped opportunity: How public-private partnerships can advance education for all. Technical report, Education Policy and Data Center, Washington, DC, 2006.

Data

On the course website[14] we have made available a data table that was extracted from the Education Policy and Data Center's (EPDC) online database of education-relevant data for developing countries (see *http://epdc.org*). As the EPDC website notes, "The data is of varying quality and varying comparability. The EPDC does not alter the data in any way." In other words, don't expect all the numbers to make sense.

[14] `http://wb/modsim/handouts`

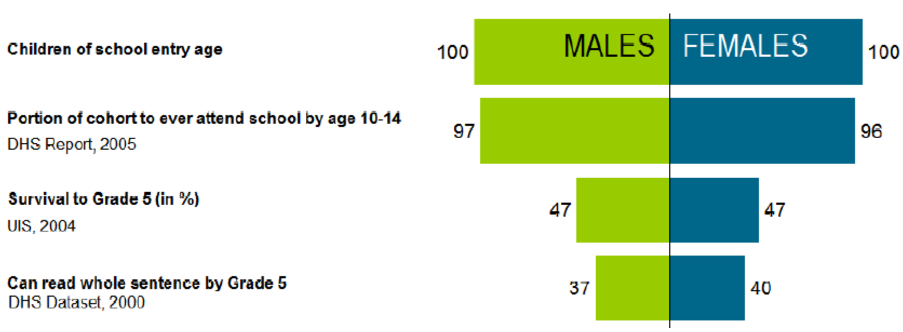

Figure 5: Learning pyramid for Uganda. The figure indicates the likelihood that a current Ugandan 6 year-old will learn to read under existing conditions (figure from *http://epdc.org*).

The provided file includes the following raw data for some or all of the years between 1999 and 2006:

Number of pupils: Number of pupils enrolled in that grade in that particular year. This number includes both pupils who were promoted from the previous grade, and students who are repeating the grade.

Number of repeaters: Number of enrolled pupils in that grade who are repeating the grade in that particular year. Note that it is possible for pupils to repeat more than once.

On-time pupils (%): Fraction of enrolled pupils of who are of the "expected" age for that grade.

Over-age pupils (%): Fraction of enrolled pupils of who are older than expected for that grade.

Under-age pupils (%): Fraction of enrolled pupils of who are younger than expected for that grade.

Both gender-specific and aggregate data are provided. Note that there is a lot of data here; you are not necessarily expected to use all of it. Much more data is available on the EPDC website, should you wish to construct a more detailed model[15].

[15] And get a summer job in Washington.

Deliverables

Your task for this case study is to create a demographic model of the primary education system in Uganda, and use it to make a recommendation about in which grade(s) investments might most sensibly be made to help Uganda achieve the goal of having all of its pupils complete fifth grade. As you are expected to make only a preliminary recommendation, you should only consider *where* to spend the money, not *how* to spend the money. In other words, you should be thinking about whether it makes more sense to make investments across the board, or whether it is a better policy to target particular grades. You should not be thinking about the choice of investing in textbooks versus in classroom space[16]. This decision is, of course, complicated by the facts that different grades have substantially different populations, and most educational investments (textbooks, classrooms, teachers) scale with the number of students.

[16] Sometimes it is best to keep things simple.

Your final deliverable for this case study should be a short memo (1-2 pages) that clearly states and justifies your recommendation regarding resource investments. The memo should be addressed to the Ugandan Minister of Education (you should assume he/she does not know MATLAB), and should include appropriately presented simulation results supporting your recommendation. This document must be turned in by 9AM on *Wednesday, September 17*.

During the course of this case study you should also produce the following intermediate deliverables:

Model Diagram and Explanation: Create a clearly annotated diagram for your model. The diagram should include a caption that describes the parameters and how you determined or will determine their values. This should be completed and posted in your workspace no later than the end of class on *Monday, September 8*.

Script: Write a MATLAB script that implements the model. The script should be well-commented, and should be simple to read. This should be completed and posted in your workspace no later than the end of class on *Wednesday, September 10*.

Validation and Preliminary Results: Create one or more figures with descriptive captions that demonstrate the extent to which your model and simulation agree with the available data. Also create one or more figures that present your preliminary results. These should be completed and posted in your workspace no later than the end of class on *Monday, September 15*.

References

R.F. Constantino, J.M. Cushing, B. Dennis, and R.A. Desharnais. Experimentally-induced transitions in the dynamic behavior of insect populations. *Nature*, 375(6528):227–230, Jan 1995.

Inter-Agency Expert Group. The millenium development goals report. Technical report, United Nations, New York, New York, 2007.

G. Ingram, A. Wils, B. Carrol, and F. Townsend. The untapped opportunity: How public-private partnerships can advance education for all. Technical report, Education Policy and Data Center, Washington, DC, 2006.