COLBY COLLEGE
Department of Mathematics
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MA332
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Assignment 9: Discrete Fourier Transform

Reading: Handouts from *Numerical Recipes* and *Physics of Music.*

Due: Tuesday 24 November

Big picture: This week you are going to experiment with the DFT and take some steps toward an implementation of the FFT. Next week we will do the real implementation of the FFT.

Play with the DFT

1. Pick up the Maple worksheet named `hw9_demo.mws` from the web page and run through it, comparing the implementation of the DFT with the description in the book. You will have to work with and modify this implementation, so read it carefully.

2. Make sure that you can answer the following questions:

   (a) What is the sampling rate? The critical frequency?

   (b) What is the meaning of the f_n? How do you interpret the horizontal axis in the figure? See page 406.

   (c) Why are the amiplitudes at the positive frequencies the same as the amplitudes at the negative frequencies?

3. You might want to change the way the DFT is plotted so that it only shows the positive frequencies, or so the horizontal axis is labelled correctly.

Play

1. Add a second frequency component to the signal, with frequency 0.2. See what the DFT looks like.

2. Try out a variety of values for the second frequency, including 0.33, 0.49, 0.6, 0.8, and 0.9.

3. Write a few paragraphs describing your experiments and what you saw. Include interesting figures. Use the word “aliasing”.

4. For one or more of these signals, check Parseval’s theorem.

Encapsulate

1. Take the code I have provided and encapsulate it in a procedure. If you are not sure of the Maple syntax for procedures, read the book. Also, check out the examples from past homework solutions. You procedure should be called dft. It should take two arguments, an array of complex numbers (the signal) and an integer indicating the length of the array. It should return an array of complex numbers (the DFT).

2. Test your procedure and make sure it yields the same result as the unencapsulated code.

Zip it up

1. Implement the transformation explained on page 408. Divide the array into evens and odds, and use the dft procedure to find the DFT of each half. Then stich the two halves into the DFT of the original array.

   HINT: the tricky thing here is that the two sub-DFTs are only half as long as the original, so you have to double them up by repeating them (or by taking the indices mod N/2).

2. Make sure you get the same result again.