COLBY COLLEGE
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Assignment 7: QR iteration

Reading: Faires and Burden Chapter 9 (but not 9.5).
Due: Thursday 5 November

Householder transformation

1. Starting with the matrix on the bottom of page 388, use Maple to perform Householder transformations, generating a tridiagonal matrix that is similar to A.
2. Use `eigenvalues` to find the eigenvalues of A and the tridiagonal matrix, and confirm that they are similar.

   Hint: Make one of the elements of the matrix a floating-point number, then `eigenvalues` will run faster and return numerical answers. Also, subsequent calculations will be done in FP without your having to put evalf's all over the place.

QR iteration

1. Use `QRdecomp` to find a QR factorization of the tridiagonal result of the previous section.
2. Multiply R by Q and examine the off-diagonal elements to see if they seem to be getting smaller.
3. Perform a few iterations of QR and comment on the apparent rate of convergence.

Shifted QR

1. Use the upper-left element of the array as a shift, z, and form the shifted matrix $\mathbf{A} - \mathbf{zI}$.
2. Perform a few QR iterations on the shifted matrix and comment on the rate of convergence. The off-diagonal element in the lower-right corner should be going to zero fast. If not, you are doing something wrong. Iterate until it is below 10^{-5}.
3. Extract the lower-right diagonal element and shift it back. Confirm that it is one of the eigenvalues of A.
4. Extract the 3×3 matrix in the upper left using `submatrix`. Calculate its eigenvalues and confirm that they are the other three eigenvalues of A.

If either of the previous steps fails, do not proceed.

Shift again

1. Shift the reduced matrix using the upper-left diagonal element.
2. Perform QR iterations until an off-diagonal element gets below 10^{-5}.
3. Extract the next eigenvalue, shift it back and confirm that it is one of the eigenvalues of A.
4. Extract the 2×2 matrix in the upper left, and confirm that its eigenvalues are the remaining eigenvalues of A.