COLBY COLLEGE
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Assignment 3: Quadrature

Reading: Faires and Burden Chapter 4.

Due: Thursday 1 October

The purpose of this assignment is to practice the various techniques for numerical integration, and to do a little Maple programming.

Basic quadrature

1. Use Maple to plot the function $f(x) = x^2e^{-x}$ on the interval (0, 2). Using Maple or a calculator, estimate the integral of $f(x)$ over this interval, using the Midpoint rule, the Trapezoid rule, and Simpson's rule.
2. Using the functions `middlesum`, `trapezoid` and `simpson` in the Maple student library, check your answers to the previous question (see page 121).
3. Use Maple to calculate the exact value of this integral (at least to 10 digits of accuracy).
4. Calculate the absolute errors of each of the estimates. Looking at the plot of the function, see if the sign of the errors make sense. For example, we expect the Trapezoid rule to underestimate the integral of any function that is concave in the interval.
5. Using Maple to differentiate and maximize (see the Maple command `minimize`), find the maximal values of $f''(x)$ and $f^{(4)}(x)$ in the interval, and use these to find a bound on the errors for each of the estimates. Confirm that the errors fall within these bounds.
6. Answer questions 9 and 10 on page 115 of the book.

Composite quadrature

1. Use the loop on page 122 of the book to evaluate the Composite Midpoint rule for the same $f(x)$, using 2, 4, and 8 intervals. Notice that unlike for the other Composite rules, the Midpoint rule uses n in a weird way that does not correspond to the number of subintervals. I suggest that you create a variable called `intervals`, and set `n := 2 * (intervals-1);`
2. Write a similar loop to evaluate the Composite Trapezoid rule. Hints: The Maple interface for editing is not as wonderful as we would like, so it might be a good idea to write your loop on paper before you type it in. Also, note that the calculation of h and the x_i is different for the Trapezoid rule, so don't just look at what's in the blue box.
3. Use your loop to evaluate the Trapezoid rule with 2, 4 and 8 intervals.

4. Make a table that shows all eight estimates (two rules, 1, 2, 4, and 8 intervals), and for each estimate, calculate the absolute error and the number of times $f(x)$ was evaluated to get the estimate (again, be careful about the fact that for the Midpoint rule n is *not* the number of subintervals. Overall, which method seems to be more efficient?

5. Use either your loop or the book's loop to answer Question 11 on page 124. Notice that the function is symmetric around zero, if that gives you any ideas.

Romberg integration

1. Use the Maple command `trapezoid` to check your answers to the previous problem, and then use the last two estimates (with 4 and 8 intervals) to generate an improved estimate by Richardson's extrapolation.

2. Calculate the absolute error of this estimate. What is the order of this error term (power of h)?

Gaussian quadrature

1. Perform Gaussion quadrature on our favorite $f(x)$, with $n = 2$ and $n = 3$.

2. Calculate the absolute errors of the estimates and compare them to the errors of the other methods, for comparable numbers of function evaluations.