COLBY COLLEGE
Department of Mathematics
and Computer Science

MA332 **Allen B. Downey**
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Assignment 2: Root-finding

Due Date: Thursday 24 September

The purpose of this assignment is to give you a chance to try out the root-finding techniques from the book on various problems, and to get some practical exposure to Maple.

Reading: Faires and Burden, Chapter 2. Skim 2.5 and 2.6. Also, Maple Programming Guide, Chapter 7.

A little arithmetic

We can use Newton's method to find the (positive) square root of a by finding the root of $x^2 - a = 0$. In this case:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{x_n + a/x_n}{2} \tag{1}$$

where x_0 is the initial guess.

1. Use Maple to find the square root of 6 using the initial guess $x_0 = 6$. Perform your calculations using 40 digits and continue until the estimate is accurate to all 40 digits.

2. For each of the x_i, calculate the absolute error (compared to the correct answer), the relative error, and the number of digits of accuracy.

NOTE: Be sure to pick up `notes_maple1.txt` from the notes directory of the class web page. It contains the info about Maple you need to do this problem.

What to turn in: I will leave it up to you to find an effective way to present the results of this exercise. You can print a Maple worksheet, or you can make a table of the results, or some combination, but you should try to keep it one one or (at most) two pages.

A little analysis

1. Show that quadratic convergence implies that the number of correct digits doubles at each iteration (see the top of page 45). When I say "show that," I mean write a sketch of a proof, keeping in mind that as a non-mathematician I have a fairly lax definition of proof.

2. What approximation did you have to make in order to derive the doubling-digit heuristic? What is the implication of that approximation.

Note: this problem is necessarily vague. Do your best.

Word problem!

1. Do exercise 15 on page 41 of the book, using Bisection, False Position, or Secant. Be sure to indicate which method you think you are using. You can use any precision arithmetic, and you can use either Maple, a calculator, or a general-purpose programming language (but you cannot use anyone else's implementation of any root-finding method).
2. Explain how you chose your initial interval. This is an often neglected step, but it is an important part of the algorithm.
3. Use Maple's fsolve command to check your answer, but don't cheat by using the correct answer to choose the initial interval.

The Maple command to solve a non-linear equation using numerical techniques is `fsolve`. If you type `?fsolve` at the Maple prompt, you will get the help page for this command, which you can print.

Another word problem!

1. Do exercise 15 on page 48 of the book using Newton's method. Your answer should be accurate to 4 significant digits. Again, you can use any precision arithmetic, and you can use either Maple, a calculator, or a general-purpose programming language (but you cannot use anyone else's implementation of any root-finding method). If you want/need to, you can use Maple to find the derivative of $f(x)$, but you should be ashamed.
2. Explain how you chose your initial value.
3. Use Maple's fsolve command to check your answer, but don't cheat by using the correct answer to choose the initial value.

SERIOUS WARNING: READ ME!

There are problems here, and on future assignments, that create a MORAL HAZARD. That is, they make it very easy for you to cheat, for example by using Maple's `fsolve` command to get the answer and then skipping the actual calculation.

There are two very important reasons you should not do this!

1. It constitutes a form of academic dishonesty. If you turn in a report that says you performed an experiment when you did not, you have committed scientific fraud, and I will treat it as such.
2. I might catch you! Keep in mind that many of the techniques we are using are conditionally stable, which means that they may or may not yield the right answer. Just because Maple gets the right answer doesn't mean you will. If I give you an ill-conditioned problem or an unstable algorithm, you had better not turn in the correct solution.